1 Let's show the Born approximation of eq. 201.1 in the lecture notes using the same logic used for the Rutherford scattering.

a Start from equation 190.2 (specialized for $1 \to 1$ scattering and $Z = 1$) and show that, for $\vec{p}' \neq \vec{p}$ the first perturbative contribution is:

$$\langle \vec{p}' | iT | \vec{p} \rangle \simeq -i \underbrace{\int_{-\infty}^{+\infty}}_{\text{ignoring the "slightly imaginary" time component here}} dt\, e^{i(E_{p'} - E_p)t} \, {}_0\langle \vec{p}' | \underbrace{\hat{H}_I}_{\text{ATTENTION}} | \vec{p} \rangle_0$$

ignoring the "slightly imaginary" time component here

ATTENTION, this is not the interaction picture operator of eq. 201.1, but the Schrödinger picture operator.

b Now, considering: $\hat{H}_I = \int d^3x \, \hat{\bar{\Psi}} \hat{\Psi} \, \underbrace{V(\vec{x})}_{\text{potencial}}$

potencial (classical object, it's a function not an operator)

show that, in the non-relativistic limit:

$$\langle \vec{p}' | iT | \vec{p} \rangle \simeq -i \tilde{V}(\vec{p}' - \vec{p}) \, (2\pi) \, \delta(E_{p'} - E_p) \underbrace{(2m \, \delta^{s'_+} \delta^{s})}_{\text{extra part}}$$

spins of the outgoing and incoming fermions, respectively

this extra part (in relation to 201.1) is due to the relativistic normalization we use here.